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LETTER TO THE EDITOR

The integer quantum Hall transition and random $su(N)$ rotation

Stanislav Boldyrev[1] **and Victor Gurarie**[2]

Institute for Theoretical Physics, Santa Barbara, CA 93106, USA

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Abstract

We reduce the problem of the integer quantum Hall transition (QHT) to a random rotation of an N-dimensional vector by using an $su(N)$ algebra, where only N specially selected generators of the algebra are nonzero. The group-theoretical structure revealed in this way allows us to obtain a new series of conservation laws for the equation describing the electron density evolution in the lowest Landau level. The resulting formalism is particularly well suited to numerical simulations, allowing us to obtain the critical exponent ν numerically in a very simple way. We also suggest that if the number of nonzero generators is much less than N, the same model, in a certain intermediate time interval, describes percolating properties of a random incompressible steady two-dimensional flow. In other words, the QHT in a very smooth random potential inherits certain properties of percolation.

1. Introduction

The quantum Hall transition (QHT) is a delocalization transition of a particle moving on a two-dimensional plane with a random potential and a strong magnetic field perpendicular to the plane [1]. Recently, an elegant approach to this transition was suggested by Sinova *et al* [2]. The idea of the method is to consider the quantum mechanical states of a particle belonging to the lowest Landau level, and to project the density operators $\hat{\rho}(x, t)$ onto these functions. The spatial correlation properties of the particle can then be described by the correlation function of these projected density matrices, $G(x, t) = \mathrm{Tr}(\hat{\rho}(x, t)\hat{\rho}(0, 0))$, or by its average, $\langle G(x, t)\rangle$. In this formula, 'Tr' corresponds to summing over all the states of the lowest Landau level, and the angular brackets denote averaging over the random potential.

In a certain sense, $G(x, t)$ is the probability of transition of a quantum particle from the origin to point x in the course of time t. This statement would be exactly true if we allowed the particle to travel over all the quantum states, i.e., not only those belonging to the lowest Landau level. Not having the full system of wavefunctions prevents us from localizing

[1] Current address: Department of Astronomy and Astrophysics, University of Chicago, Chicago, IL 60637, USA.
[2] Current address: Theoretical Physics, Oxford University, 1 Keble Road, Oxford OX1 1JP, UK.

the particle at a distance smaller than the magnetic length $l^2 = \hbar c/(eB)$. For example, the initial condition for the G-function describing a particle placed at the origin has the form $G(x, 0) = A \exp(-x^2/2l^2)$, with some normalization constant A. However, for strong enough magnetic fields, the magnetic length l is much smaller than the size of the system and particles can be adequately localized.

The projected density operators obey the Schrödinger equation

$$\mathrm{i}\hbar \frac{\partial}{\partial t}\hat{\rho} = [\hat{H}, \hat{\rho}], \tag{1}$$

whose Hamiltonian is just the random potential $V(x)$ projected onto the lowest Landau level, $\hat{H} = \int \mathrm{d}^2x\, V(x)\hat{\rho}(x)$. The correlation function G satisfies an analogous equation, which due to remarkably simple commutation relations between the projected density operators $\hat{\rho}$, can be written in the Fourier space ($\boldsymbol{k}$ is a two-dimensional vector, $\boldsymbol{k} = (k_1, k_2)$) as [2]

$$\mathrm{i}\hbar \frac{\partial}{\partial t}G(k, t) = \int \mathrm{d}^2q\, 2\mathrm{i} \sin\left(\frac{l^2}{2}\boldsymbol{k} \times \boldsymbol{q}\right) V(k - q) \exp\left[-\frac{l^2}{2}(k^2 - \boldsymbol{k} \cdot \boldsymbol{q})\right] G(q, t). \tag{2}$$

For a simple, 'from first principles', derivation of this equation we refer the reader to [3]. All the information about the QHT is contained in this equation, which we use as a starting point in the present work (see also [4]).

Unfortunately, it is not obvious how to solve this equation analytically. However, it is physically clear that only N degrees of freedom should be relevant in the evolution described by (2), where N is the number of states in the lowest Landau level. It must therefore be possible to reduce the infinite-dimensional equation (2) to an N-dimensional dynamical system preserving all the physics of the QHT. Such a reduction would also be cost-effective for numerical calculations of the localization exponent on the basis of equation (2). A simple introduction of a cut-off in the k-space cannot be satisfactory, since it breaks the symmetry of the system. In this letter, we demonstrate another, rather effective, way of excluding the irrelevant degrees of freedom. We perform the reduction by revealing the hidden group-theoretical structure of equation (2). We reduce the problem of the QHT to a finite-dimensional problem of a random rotation by banded $su(N)$ matrices with the bandwidth $n \sim \sqrt{N}$. We show that equation (2) admits a series of conserved integrals of motion that were not known before, and that are preserved in our model. We also show that when the width of the band, n, is much smaller than $\sqrt{N}$, our model describes classical percolation.

Let us start with the formal classical limit $l \to 0$. In this case, equation (2) is simplified considerably:

$$\frac{\partial}{\partial t}G(x, t) = \epsilon_{ij}\, \partial_i V(x)\, \partial_j G(x, t). \tag{3}$$

A formal solution to (3) can be written at once [3]. It is $G(x, t) = \delta(x - x(t))$ where $\frac{\mathrm{d}x_i}{\mathrm{d}t} = \epsilon_{ij}\, \partial_j V(x)$, equations which were thoroughly studied in [5]. From here it is easy to deduce that we are trying to describe a particle which percolates along the equipotential lines of a random potential. Physically, this quasiclassical limit describes a slow drift of the guiding centre of a little Larmor circle, around which the particle is rotating fast in a strong magnetic field. This drift occurs due to the force $\boldsymbol{F}(x) \sim \boldsymbol{\nabla} V(x)$, and its velocity is $\boldsymbol{v}(x) = \boldsymbol{F}(x) \times \boldsymbol{B}$. Obviously that does not capture the physics of the QHT. The apparent paradox is resolved if we note that the Larmor circle cannot be made smaller than l, and will therefore be eventually destroyed.

The reason that a solution to (3) was so easy to write down lies in its infinitely many integrals of motion. Indeed, it is not hard to check that any expression of the form

$$I_{mn} = \int \mathrm{d}^2x\, G^m V^n \tag{4}$$

is conserved along the solutions of (3). The conservation of I_{10} and I_{11} is simply a consequence of the conservation of probability and energy. Higher-order integrals are the consequence of incompressibility of the velocity field $v(x)$. Now, in a quite remarkable way, these integrals of motion do not get destroyed as one goes back to the original equations (2), at least for integer m and n. They only get slightly modified.

To see that, let us put the particle on a torus. The wavefunctions of the lowest Landau level can be written explicitly in the Landau gauge:

$$\psi_\alpha(x, y) = \left[\sum_m \exp\left(2\pi(x + \mathrm{i}y)(Nm + \alpha) - \frac{(Nm + \alpha)^2}{N}\pi\right)\right] \mathrm{e}^{-\pi N x^2}. \tag{5}$$

Here $0 \leqslant x, y \leqslant 1$, N denotes the number of flux quanta through the torus, and α goes from 0 to $N - 1$, labelling the N states on the torus in the lowest Landau level. These wavefunctions describe the electron localized along a narrow strip around the line $x = \alpha/N$. Notice that the magnetic length l is now automatically chosen in the form

$$l^2 = \frac{1}{2\pi N}. \tag{6}$$

It is now a matter of simple calculation to project the density operators $\hat{\rho}_{(k_1,k_2)} \equiv \exp(2\pi\mathrm{i}(k_1 x + k_2 y))$ onto the lowest Landau level on the torus. In this notation, k_1 and k_2 are integers. As a result, the density operators are now $N \times N$ matrices in the basis of states (5), which can be written in the following form.

Consider a pair of unitary unimodular $N \times N$ matrices:

$$h = \begin{pmatrix} 0 & 1 & \cdots & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & \cdots & \cdots & \cdots & 0 \end{pmatrix}, \tag{7}$$

$$f = \mathrm{diag}(1, \epsilon, \ldots, \epsilon^{N-1}), \tag{8}$$

where h is a cyclic permutation matrix, and $\epsilon = \exp(2\pi\mathrm{i}/N)$. These matrices have the following properties: $hf = \epsilon fh$, $h^N = f^N = 1$. Now introduce the matrices

$$L_{(k_1,k_2)} = \epsilon^{k_1 k_2/2} f^{k_1} h^{k_2}, \tag{9}$$

where $(k_1, k_2) \neq (0, 0)$. The matrices $L_{(k_1,k_2)}$ are periodic in k_1 and k_2 with period N up to coefficient ± 1. They can be chosen as a basis for the $su(N)$ algebra. The product and the commutator of two such operators have the following forms:

$$L_q L_p = \exp\left(\frac{\pi\mathrm{i}}{N} \boldsymbol{q} \times \boldsymbol{p}\right) L_{q+p}, \tag{10}$$

$$[L_q, L_p] = 2\mathrm{i} \sin\left(\frac{\pi\mathrm{i}}{N} \boldsymbol{q} \times \boldsymbol{p}\right) L_{q+p}. \tag{11}$$

One can easily write down the explicit expression for the matrix elements of these matrices:

$$(L_{(k_1,k_2)})_{\alpha,\beta} = \epsilon^{k_1 k_2/2 + k_1(\alpha-1)} \delta_{\alpha,\beta-k_2|_{\mathrm{mod}\, N}}. \tag{12}$$

Then the density operator can be expressed in terms of these matrices as

$$\hat{\rho}_{(k_1,k_2)} = \exp\left(-\frac{k_1^2 + k_2^2}{2N}\pi\right) L_{(k_1,k_2)}. \tag{13}$$

To be specific, equation (13) gives the density operator in the Schrödinger representation, as opposed to (1) where the Heisenberg representation was assumed. From this point on, we will understand $\hat{\rho}$ as a time-independent density operator.

It is not difficult to check that the density operator written in this form does indeed satisfy the commutation relations discussed in the literature (for example, see [2] and references therein) with the magnetic length chosen according to (6).

The projected Hamiltonian takes the form

$$\hat{H} = \sum_{k_1 k_2} V(-k_1, -k_2)\hat{\rho}_{(k_1,k_2)}. \tag{14}$$

Since the Fourier components of the potential $V(x)$ are random, we see that the Hamiltonian is none other than a random $su(N)$ matrix. This completes the projection to the lowest Landau level on the torus. At this point we can easily rederive (2) simply by writing $\hat{\rho}$ as a matrix and commuting it with the Hamiltonian $\hat{H}$ as prescribed in (1) using the explicit matrix definitions (13). Finally, let us introduce the matrix $\hat{G} = \sum_{k_1 k_2} g(k,t)L_{(k_1,k_2)}$, where $g(k) = G(k)\exp(k^2 l^2/4)$. Here as a consequence of choosing k_1 and k_2 on the torus as integer numbers, we use the notation $k = 2\pi(k_1, k_2)$. As a consequence of (2), it satisfies an equation identical to (1):

$$\mathrm{i}\hbar\,\frac{\partial}{\partial t}\hat{G} = [\hat{H}, \hat{G}]. \tag{15}$$

Through all these manipulations we succeeded in reducing the original problem of a particle on an infinite plane in a random potential to a problem with a finite number of degrees of freedom. That allows us immediately to write down the generalizations of the integrals of motion (4). Indeed, since a trace of a commutator of finite matrices is equal to zero,

$$\tilde{I}_{mn} = \mathrm{Tr}(\hat{G}^n \hat{H}^m) \tag{16}$$

is conserved. These integrals can be expressed in terms of $G(k,t)$ and $V(k)$ using explicit expressions for the matrices $\hat{\rho}$.

The first few integrals in (16) coincide with their 'classical' counterparts in (4). For example, $\tilde{I}_{10}$ and $\tilde{I}_{11}$ are equal to I_{10} and I_{11} and are still the probability and energy conservation, respectively. Higher-order integrals of motion become increasingly more complicated. For example,

$$I_{30} = \sum_{k,s,r} g(k)g(s)g(r-k-s)\exp\left[\frac{\pi\mathrm{i}}{N}(\boldsymbol{k}\times\boldsymbol{s} + \boldsymbol{k}\times\boldsymbol{r} + \boldsymbol{s}\times\boldsymbol{r}) + \pi\mathrm{i}N r_1 r_2\right]. \tag{17}$$

Here $\boldsymbol{k} = 2\pi(k_1, k_2)$, $\boldsymbol{s} = 2\pi(s_1, s_2)$, $\boldsymbol{r} = 2\pi N(r_1, r_2)$, $g(k) = G(k)\exp[-\pi(k_1^2+k_2^2)/(2N)]$, and the summation is over integer k_1, k_2, s_1, s_2, r_1, and r_2.

Despite being rather complicated, this expression reduces to its classical counterpart, $I_{30} = \int \mathrm{d}^2x\, G^3(x)$, in the limit $l \to 0$ ($N \to \infty$). It is of course possible to show, after some algebra, that it is indeed conserved under the time evolution (2). A simple mathematical feature lies behind this: in the limit $N \to \infty$, the operators $\hat{\rho}$ become the generators of the group of volume-preserving diffeomorphisms on a torus [6, 7]. Such a group represents motion of an incompressible fluid, which is precisely the meaning of (3), with $V(x)$ being the so-called stream function.

2. Quantum Hall transition

We are going to demonstrate now that all the features of the QHT are preserved in this picture. Consider a particle placed in one of the states (5). That means that it is localized along the x-direction and is extended along the y-direction. Wait some time t and measure the average square of the displacement of the particle along the x-direction:

$$\langle x^2(t)\rangle \propto \frac{1}{N^3}\sum_{\alpha,\beta=1}^{N} |(\mathrm{e}^{(\mathrm{i}/\hbar)\hat{H}t})_{\alpha\beta}|^2(\alpha-\beta)^2, \tag{18}$$

where $(\ )_{\alpha\beta}$ denotes the matrix element of the matrix inside the brackets.

We expect, following [2, 3], the deviation $\langle x^2(t)\rangle$, at large enough times but before the finite size of the system is reached, to behave as

$$\langle x^2\rangle \sim t^{1-1/(2\nu)}. \tag{19}$$

The critical exponent ν is to be determined, and is believed to be close to $7/3$. To explain equation (19), note that in the lowest Landau level band, the localization length $\xi(E)$ diverges as the particle energy approaches the central energy of the band, $\xi(E) = |E - E_c|^{-\nu}$. We require that for $E = E_c$ the particle undergo a normal diffusion, $\langle x^2\rangle_c \sim t$, to a very large distance, but for $E \neq E_c$, the diffusion must occur only until $x \sim \xi(E)$; after that the particle gets localized at the localization length $x \sim \xi(E)$. For a given t, the number of energy levels ΔE contributing to the diffusion is thus $\Delta E^{-2\nu} \sim t$, which leads to the anomalous diffusion law $\langle x^2\rangle \sim t\,\Delta E \sim t^{1-1/(2\nu)}$.

The Hamiltonian (14) appears to be a linear combination of $su(N)$ matrices with random coefficients. However, if it were indeed a random $su(N)$ matrix with probability distribution *invariant* under $su(N)$ rotations, it would lead to a particle instantaneously hopping all over the torus. In fact, such a Hamiltonian would be unphysical. As a consequence of the strong exponential suppression of high Fourier modes in the density operators (13), the $su(N)$ rotation-invariant Hamiltonian corresponds to the random potential $V(x)$ varying considerably at distances much smaller than the magnetic length. In fact, in the continuum limit it would lead to the potential having Fourier harmonics growing exponentially at large k, as follows from the analysis of (13).

A more physical setting involves a potential whose Fourier harmonics at least do not grow at large k. While it is possible to choose a particular random $V(x)$ exhibiting this property and compute the Hamiltonian (14), it is not necessary to do so. Instead, we can just point out that the exponential in (13) strongly suppresses high Fourier harmonics of the random potential, such that $k_1, k_2 > n = \sqrt{2N/\pi}$. Therefore, we can simply choose the Hamiltonian to be

$$\hat{H} = \sum_{k_1,k_2=-n}^{n} v_{(k_1,k_2)} L_{(k_1,k_2)}, \tag{20}$$

where $v_{(k_1,k_2)}$ are random independent variables with equal mean square values $\langle v^2\rangle$. The crucial part of (20) is that the Hamiltonian is a linear combination of only N generators of the $su(N)$ algebra out of a total of N^2 generators, with random coefficients. From the explicit form (12) of matrices $L_{(k_1,k_2)}$, we see that the random matrix $\hat{H}$ has nonzero elements only on the diagonal strip of width $\sim N^{1/2}$. Within the strip, only the matrix elements no further than $\sim N^{1/2}$ from each other along a given diagonal are correlated with each other. Such *banded random matrix* theory, which reproduces the physics of the QHT, was considered in a similar context in [8, 9], although the Hamiltonian (20) was not introduced there. We emphasize that the geometry of the torus was particularly convenient in helping us to arrive at (20), but was not at all necessary. Equation (19) is true only for moderate times t when the particle would not have reached the boundary of the system, and so the geometry of the sample is irrelevant.

This framework provides a very convenient setting for numerical calculation of ν. Below we present the numerical simulations for a 1000×1000 matrix. The Hamiltonian has been chosen in the form (20). The random coefficients $v_{(k_1,k_2)}$ have thus been generated in the square $|k_1|, |k_2| < n = \sqrt{2N/\pi}$, and the real and imaginary parts of each Fourier mode $v_{(k_1,k_2)} = v^*_{(-k_1,-k_2)}$ were chosen randomly, independently, and uniformly from the interval $[-0.5, 0.5]$. Instead of calculating the matrix exponent in (18) directly, we have simulated the

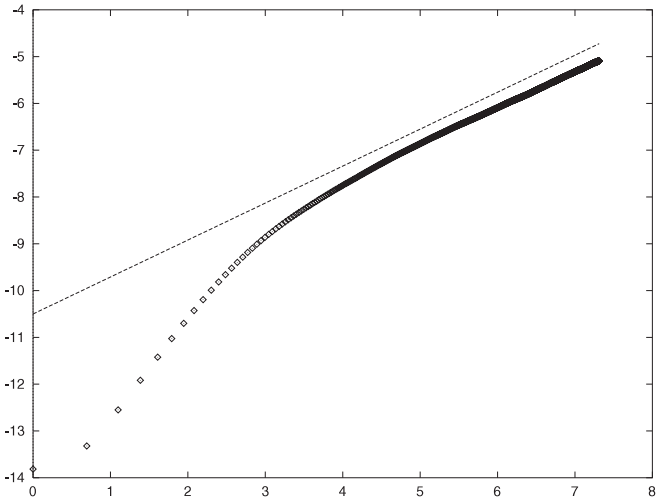

Figure 1. $\log(\langle x^2 \rangle)$ is plotted as a function of $\log(t)$ for $N = 1000$, $n = 15$. The light line has the slope 0.79 which corresponds to $\nu = 2.38$.

equation

$$\frac{\partial}{\partial t} Z_\alpha = \mathrm{i} \sum_\beta \hat{H}_{\alpha\beta} Z_\beta, \tag{21}$$

where Z_α is a wavefunction in the representation of the states on the torus. It is easy to check that in this representation,

$$g(k_1, k_2) = Z^*_\alpha [L_{(k_1,k_2)}]_{\alpha\beta} Z_\beta, \tag{22}$$

where $g(k)$ was introduced in (17).

After the random potential $\hat{H}$ had been generated, and the initial distribution had been chosen in the form

$$Z_\alpha = \begin{cases} 1, & \text{if } \alpha = \alpha_0, \\ 0, & \text{otherwise,} \end{cases} \tag{23}$$

the dispersion $\langle x^2(t) \rangle = \sum_\alpha (\alpha - \alpha_0)^2 |Z_\alpha|^2$ was calculated as a function of time. The same calculation was performed for all the initial positions of the particle, $\alpha_0 = 1, \ldots, N$, and the average was taken over all such realizations. The result is shown in figure 1. As we see, we closely reproduce the universally accepted value of ν without much difficulty.

3. Discussion and conclusions

As was already discussed, a naive $l \to 0$ limit leads to the percolation picture of the QHT. However, $l \to 0$ is the same as $N \to \infty$. It should indeed be possible to reproduce the percolation behaviour if the scale of the random potential is much larger than the magnetic length l, but much smaller than the size of the system. In other words, we need to keep only those modes in (14) for which $1 \ll n \ll N^{1/2}$. The percolating behaviour holds until the width of the diagonal strip of the matrix $\exp(\mathrm{i}\hat{H}t)$ becomes equal to $N^{1/2}$. After that, the regime changes to the QHT. The square of this width is given by (18), and therefore grows as $t^{1-1/(2\nu^*)}$, with the percolation ν^* equal to $4/3$. The crossover time (mixing time) is thus equal,

in physical units, to $V_0 t_m/\hbar \sim (N/n^2)^{2\nu^*/(2\nu^*-1)} \sim (l_0/l)^{4\nu^*/(2\nu^*-1)}$, where l_0 is the scale of the random potential and V_0 is its typical amplitude. The percolation (or classical) regime is valid for $t < t_m$. We stress that the mixing time was estimated for the *second moment* of the deviation; in general, different moments may exhibit different mixing times, due to the multifractal character of the eigenstates with $E \sim E_c$.

Our simulations of equation (21) show that for $n \leqslant (2N/\pi)^{1/2}$, the behaviour of x^2 coincides with (19) at $t \to \infty$, but follows some intermediate asymptotics before that. We expect these asymptotics to be identical to those describing steady percolating flow of an incompressible two-dimensional fluid, i.e., $\nu^* = 4/3$. This seems to agree with our numerics, but more extensive simulations are required. In the other limit, $n \gg \sqrt{2N/\pi}$, the behaviour becomes diffusive; $\langle \alpha^2 \rangle \sim t$.

It is instructive to directly observe from the formulae (21) and (12) how the quasiclassical transition is made in the limit $n \ll N^{1/2}$. Let us keep n fixed and formally take the limit $N \to \infty$. Then introducing a new function $\psi(x,t) \equiv Z(\alpha = xN/2\pi, t)$, and using the notation $2\pi/N \to a = 4\pi^2\hbar c/(eB)$, we get the following path-integral representation for the wavefunction $\psi(x,t)$:

$$\psi(x,t) = \int^{q(t)=x} \mathcal{D}q\, \mathcal{D}p \exp\left\{ \frac{\mathrm{i}}{a} \int_0^t [p\dot{q} - V(p,q)]\, \mathrm{d}t \right\}, \tag{24}$$

where $V(x_1, x_2)$ is a random potential in a configuration space, and the symmetric Weyl time discretization (ordering of $\hat{p}$ and $\hat{q}$) is implied. We thus obtain that the classical trajectories simply correspond to percolations in a two-dimensional phase space of one-dimensional quantum mechanics with Hamiltonian $V(p,q)$! However, if we carefully follow the transition from (21) and (12) to (24), we observe that the variables p and q are both discretized with a step that is $\sim\sqrt{a}$. We are not allowed to set this spatial cut-off to zero independently of $\hbar$, which leads to breakdown of the quasiclassical solution at large times. This mechanism resembles that of the appearance of anomalies in quantum field theories.

In real experiments, the random potential length is often much larger than the magnetic length l. It would be interesting to devise an experiment which would probe the intermediate percolation asymptotics, perhaps by looking at finite-frequency conductivity.

In conclusion, we have presented a model that reduces the problem of the QHT to a finite-dimensional problem of random rotation by $su(N)$ matrices. As a consequence, equation (2) describing the QHT admits a series of integrals of motion. The random rotation matrix $\hat{H}$ is quite arbitrary, except for the dependence on the parameter N through the exponential cut-off in (13). This rather crucial dependence amounts to the vanishing of all the matrix elements but those belonging to a diagonal strip of width $n \sim N^{1/2}$. Within this framework, the QHT exponent ν can easily be evaluated numerically. Attempts to change n independently of N lead to other models, e.g. those describing diffusion and percolation in a steady incompressible 2D flow. That leads to a prediction that in very smooth potentials the QHT should exhibit some properties of percolation.

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showed that an equation similar to (2) can be formulated for *any* quantum mechanical system, not only for a particle in a magnetic field projected to the lowest Landau level.
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